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Neural-Symbolic Integration: A Compositional Perspective

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State of the art

- **Imposes restrictions on the syntax and the semantics of the logical theories:**
  - [5] translates acyclic or propositional theories to neural networks.
  - [6] replaces logical computations by differentiable functions.
  - [2,4] adopt theories with interpretations taking continuous values, e.g., fuzzy logic, probabilistic logic.
- **Depends on the semantics and the complexity of the specific theory.**

Objectives

- **Develop a compositional framework in which users can plug in *any* logical theory and *any* neural component of interest.**

- **Benefits:**
  - Control the inference cost.
  - Control the expressive power of the theory (e.g., support for non-monotonic theories not supported by PLP-based neural-symbolic frameworks as [2]).
  - Support for techniques coming from the learning theory community (e.g., implicit learning [9]).

Contributions

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- **A framework supporting those properties [1].**

- **Beyond the benefits mentioned before, compositionality allows integrating in a natural way the predictions of the neural component during the training process as opposed to prior art, e.g., [2].**

- **Compositionality is achieved via symbolic modules offering the following interfaces:**
  - *deduction*, or forward inference; and
  - *abduction*, through which one computes (i.e., abduces) the inputs to the symbolic module that would d educe a given output.

Setting

- **Integrate a symbolic module adopting a theory T and computing a function $s(\cdot)$ on top of a neural module computing a function $n(\cdot)$.**

Neural module: $n: \mathcal{X} \rightarrow [0,1]^k$

Translator: $r: [0,1]^k \rightarrow \mathcal{A}$

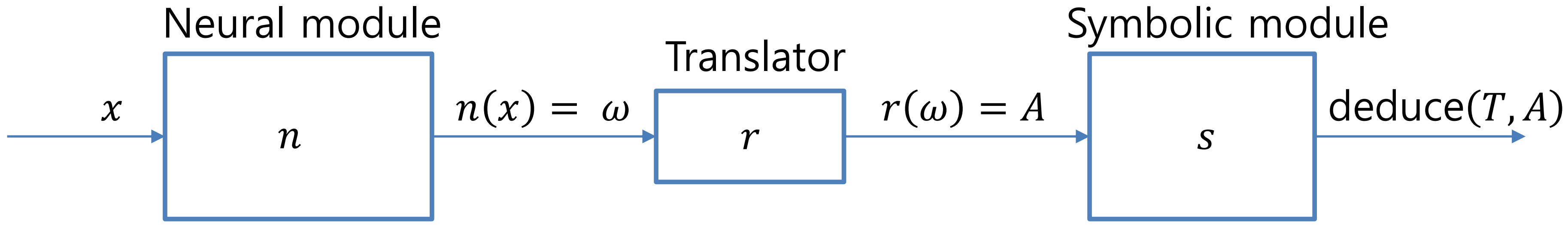
Symbolic module: $s: \mathcal{A} \rightarrow \mathcal{O}$

$$\rightarrow \boxed{n: \mathcal{X} \rightarrow [0,1]^k} \rightarrow \boxed{r: [0,1]^k \rightarrow \mathcal{A}} \rightarrow \boxed{s: \mathcal{A} \rightarrow \mathcal{O}} \rightarrow$$

- **The translator respects the semantics of the theory, e.g., if T is probabilistic, then each fact is provided along with its confidence/probability.**

- **Assumptions:**
  - closed-world assumption;
  - the semantics of the **neural** outputs is known.

Setting: inference



Setting: training

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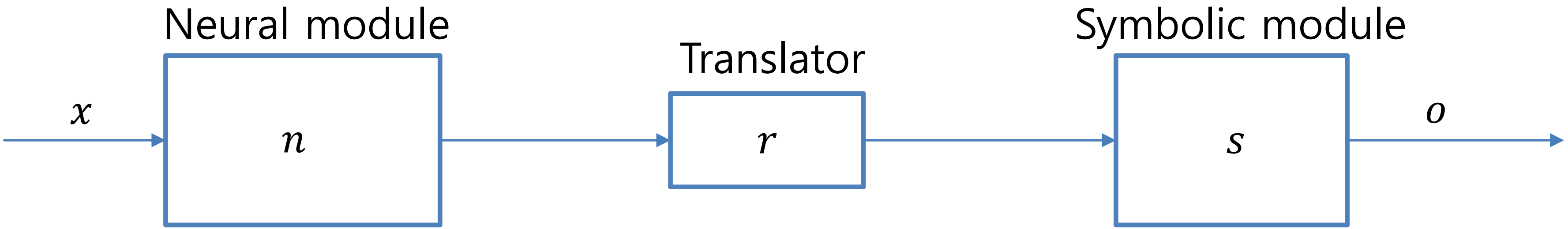
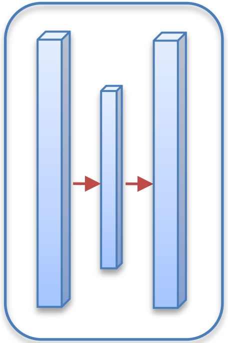
- Goal: given training samples of the form (x, o), train the *neural component.*



Example: chess



```
safe :- placed(Z1), movable(Z1).
draw :- placed(Z1), \+attacked(Z1), \+movable(Z1).
mate :- placed(Z1), attacked(Z1), \+movable(Z1).

placed(Z1) :- pos(Z1), at(b(k),Z1), pos(Z2), pos(Z3), Z2\=Z3,
              piece(w(P2)), at(w(P2),Z2), piece(w(P3)), at(w(P3),Z3).

movable(Z1) :- pos(Z2), reached(Z2,k,Z1), \+attacked(Z2).
attacked(Z2) :- pos(Z3), piece(w(P)), at(w(P),Z3), reached(Z2,P,Z3).

reached((X,Y),k,(PX,PY)) :- abs(X,PX,DX), 1>=DX, abs(Y,PY,DY),
                            1>=DY, sum(DX,DY,S), 0<S.
reached((X,Y),q,(PX,PY)) :- reached((X,Y),r,(PX,PY)).
reached((X,Y),q,(PX,PY)) :- reached((X,Y),b,(PX,PY)).
...

ic :- piece(P), at(P,Z1), at(P,Z2), Z1\=Z2.
ic :- piece(P1), piece(P2), at(P1,Z), at(P2,Z), P1\=P2.
ic :- at(b(k),Z1), at(w(k),Z2), reached(Z1,k,Z2).
ic :- piece(b(P1)), at(b(P1),Z1), piece(b(P2)), at(b(P2),Z2), Z1\=Z2.
ic :- piece(w(P1)), at(w(P1),Z1), piece(w(P2)), at(w(P2),Z2),
      piece(w(P3)), at(w(P3),Z3), Z1\=Z2, Z2\=Z3, Z3\=Z1.
```

safe

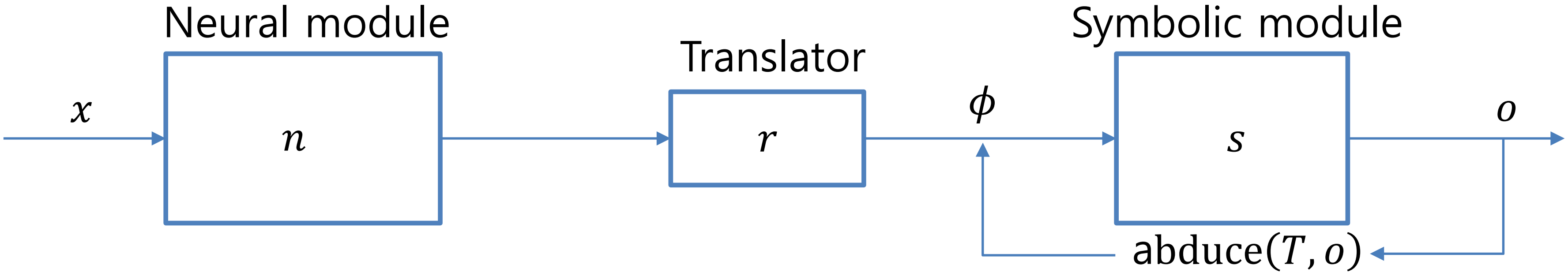
- Given an image of a chessboard and the status of the black king, learn the weights of t he neural component.

Training: high-level idea

- **Given the target label o compute a formula representing what the neural component should output in order to get the desired output after reasoning.**

- **The computation of the formula is done via *abduction***

- **Use the computed formula to train the neural component.**



Training: how do the training formulas look like?

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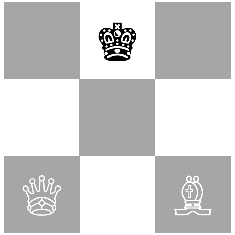
- If we want the output to be safe, the logical component should be provided with the f ollowing chessboards:

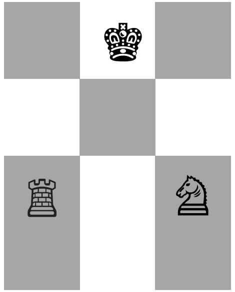
at(b(k), (2,3))

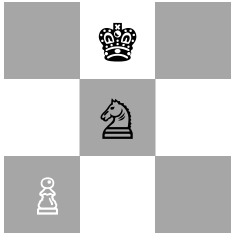


at(empty, (3,2))

at(w(q), (1,1))

at(w(b), (3,1))





at(b(k), (2,3)) ∧ **at**(w(q), (1,1)) ∧ **at**(w(b), (3,1)) ∧ **at**(empty, (1,2)) ∧ … ∧ **at**(empty, (3,2))

at(w(b), (2,3)) ∧ **at**(w(r), (1,1)) ∧ **at**(w(n), (3,1)) ∧ **at**(empty, (1,2)) ∧ … ∧ **at**(empty, (3,2)) ∨

at(w(b), (2,3)) ∧ **at**(w(p), (1,1)) ∧ **at**(w(n), (2,2)) ∧ **at**(empty,(1,2)) ∧ … ∧ **at**(empty, (3,2)) ∨

Abduction

- **Given:**
  - a set of rules P
  - a set of abducible predicates A– data that is given as part of the input to the theory–
  - a set of integrity constraints IC
  - a user query Q

find a formula Δ over of facts over A, such that

- $P \cup \Delta \vDash Q$
- $P \cup \Delta \vDash \mathrm{IC}$

Training the neural component using formulas

- The loss function must show how close –*semantically*– are the outputs of the nets to th e formula we found via abduction.
- We use *weighted model counting [11]*.

Weighted model counting

- Consider a propositional formula ϕ, where each variable X in ϕ is associated with a weight w(X) in [0,1].

- A *satisfying assignment* σ of ϕ is a mapping of the variables in ϕ to $\top$ or $\bot$, that makes ϕ true.

- The weight of a satisfying assignment σ is defined as

$$\prod_{X \in \phi | X = \top} w(X) \times \prod_{X \in \phi | X = \bot} 1 - w(X)$$

- The weighted model count of ϕ is the *sum of the weights of all* satisfying assignments of ϕ.

Weighted model counting

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$\phi = X \vee \neg Y$
$w = \{X \mapsto 0.9, Y \mapsto 0.1\}$

X	Y	ϕ	Weight of assignment
0	0	1	$(1 - w(X)) \times (1 - w(Y)) = 0.1 \times 0.9$
0	1	0	
1	0	1	$w(X) \times (1 - w(Y)) = 0.9 \times 0.9$
1	1	1	$w(X) \times w(Y) = 0.9 \times 0.1$

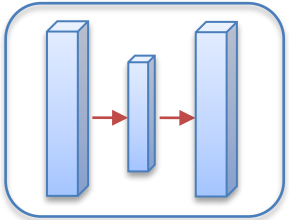
$\phi = X \vee \neg Y$
$w = \{X \mapsto 0.1, Y \mapsto 0.9\}$

X	Y	ϕ	Weight of assignment
0	0	1	$(1 - w(X)) \times (1 - w(Y)) = 0.9 \times 0.1$
0	1	0	
1	0	1	$w(X) \times (1 - w(Y)) = 0.1 \times 0.1$
1	1	1	$w(X) \times w(Y) = 0.1 \times 0.9$

Training: an example

- **Consider the formula at(b(k), (2,3)) ∧ at(w(q), (1,1)) ∧ at(w(b), (3,1)).**

cell (2,3)

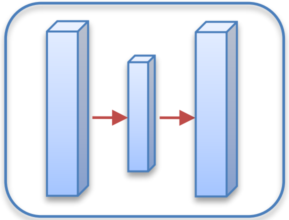


$\boxed{\text{at(b(k), (2,3))} \mapsto X_1}$

at(w(q),(2,3)) $\mapsto X_2$

at(w(b),(2,3)) $\mapsto X_9$

cell (1,1)

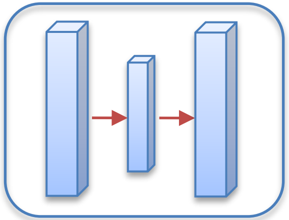


at(b(k), (1,1)) $\mapsto Y_1$

$\boxed{\text{at(w(q),(1,1))} \mapsto Y_2}$

at(w(b),(1,1)) $\mapsto Y_9$

cell (3,1)



at(b(k), (3,1)) $\mapsto Z_1$

at(w(q),(3,1)) $\mapsto Z_2$

$\boxed{\text{at(w(b),(3,1))} \mapsto Z_9}$

- Virtually create one network for each cell
- Associate each net output with a unique Boolean variable.
- The formula becomes $X_1 \wedge Y_2 \wedge Z_9$
- Set the weight of each net output as the weight of the corresponding Boolean vari able.
- The loss is the negative logarithm of the weighted model count of $X_1 \wedge Y_2 \wedge Z_9$.

Training: overview

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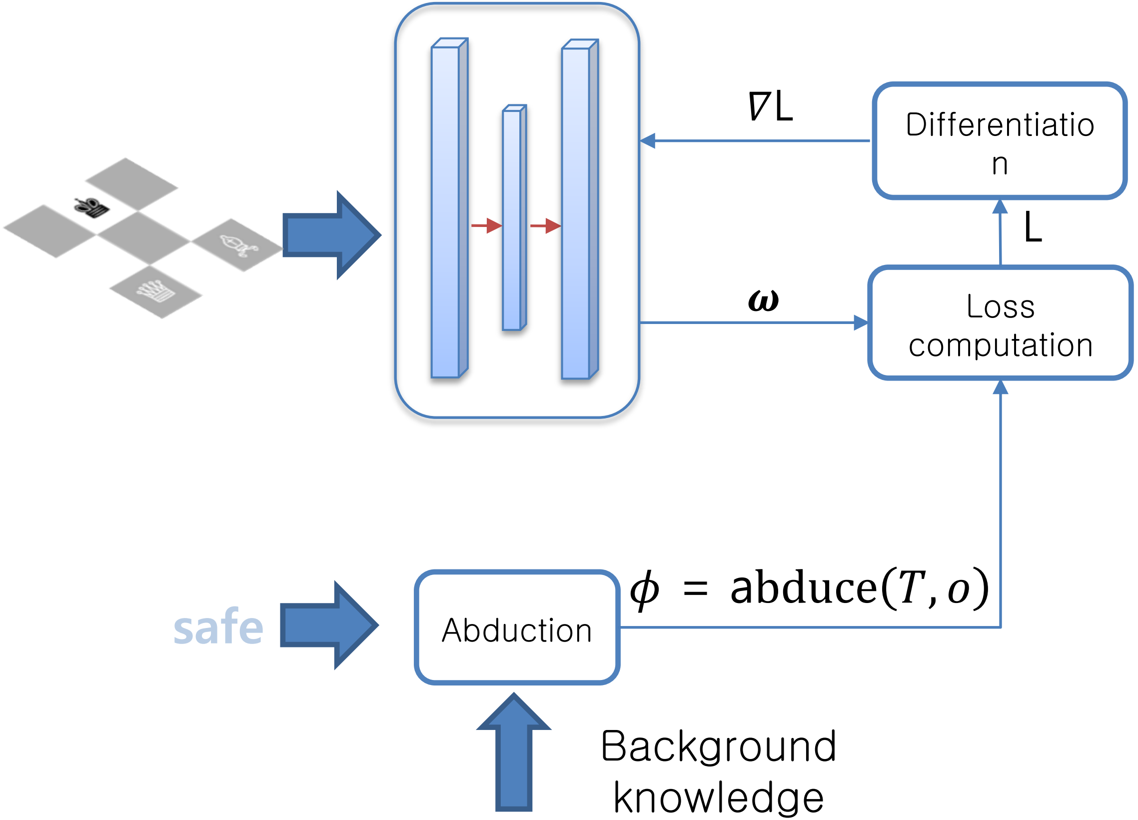
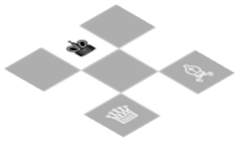
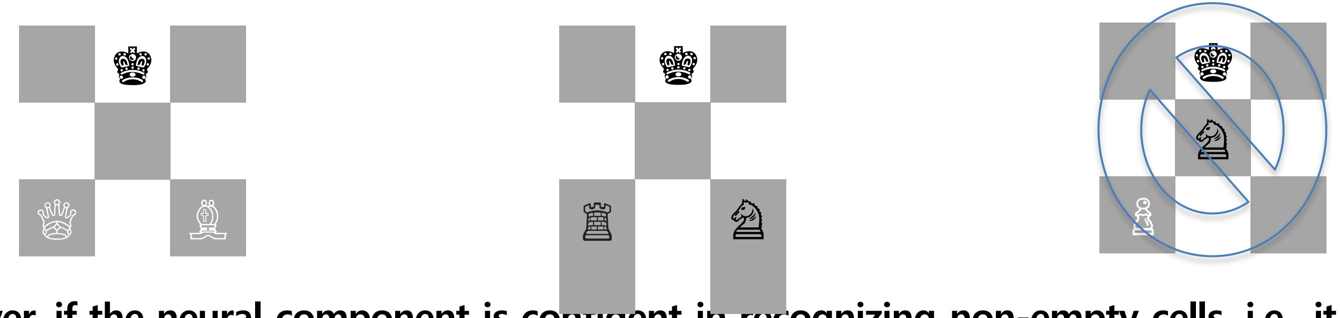


Training: neural-guided abduction

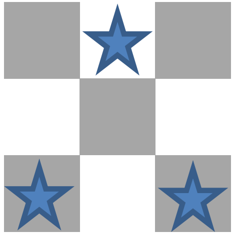
- Abduction was done so far based only on the target label.
- We could consider the neural predictions to narrow down the abductive proofs.
- Benefits: improve training efficiency.

Neural-guided abduction: example

- Recall that when provided with the training pair , safe the proofs were computed b
ased only on the training label (i.e., safe):



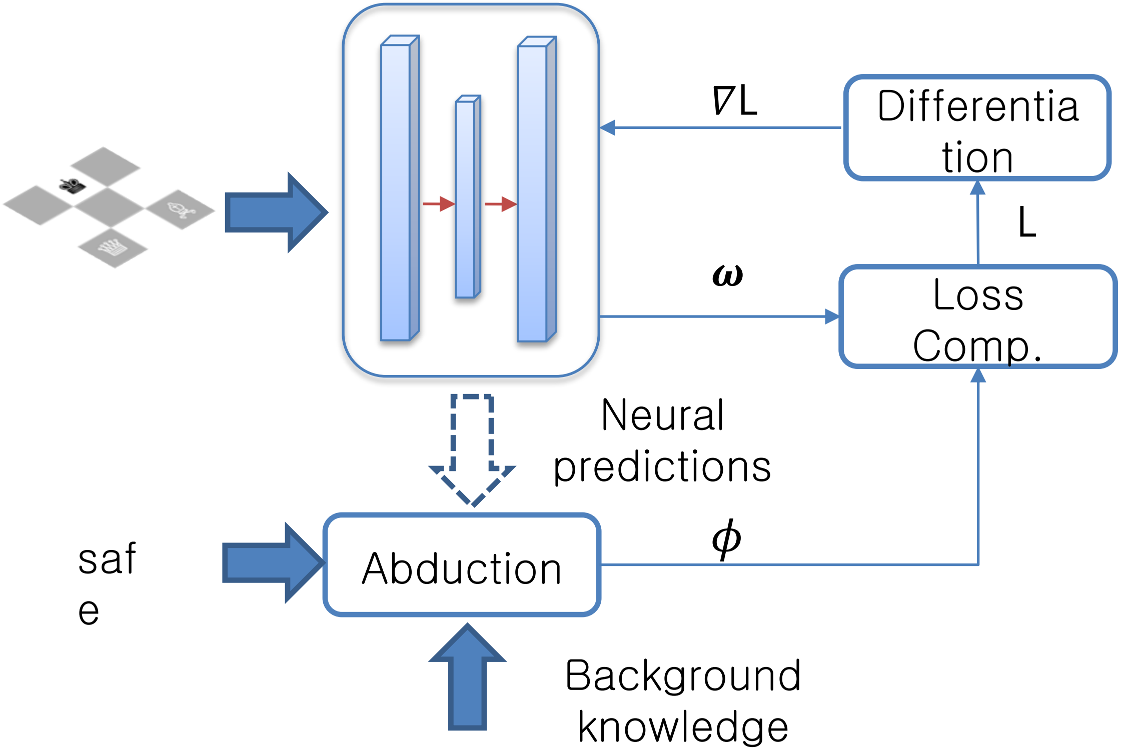
- However, if the neural component is confident in recognizing non-empty cells, i.e., it “sees”:



then we can exclude all the abductive proofs not abiding this pattern.

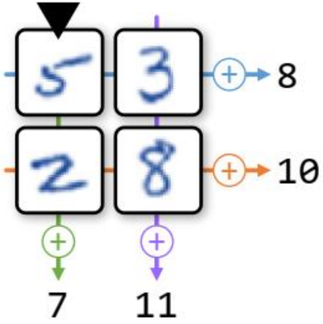
Neural-guided abduction: extensions

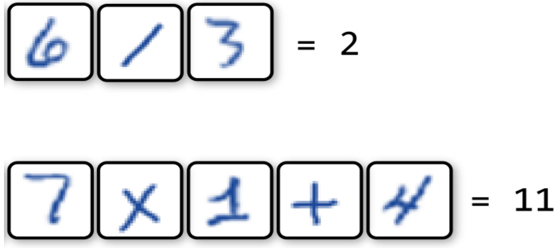
- **To support neural-guided abductio n, we need to:**
  - establish a communication channe l between the neural and the logic al components;
  - extend abduction to deal with nois y or inconsistent neural predictions via proximity functions.

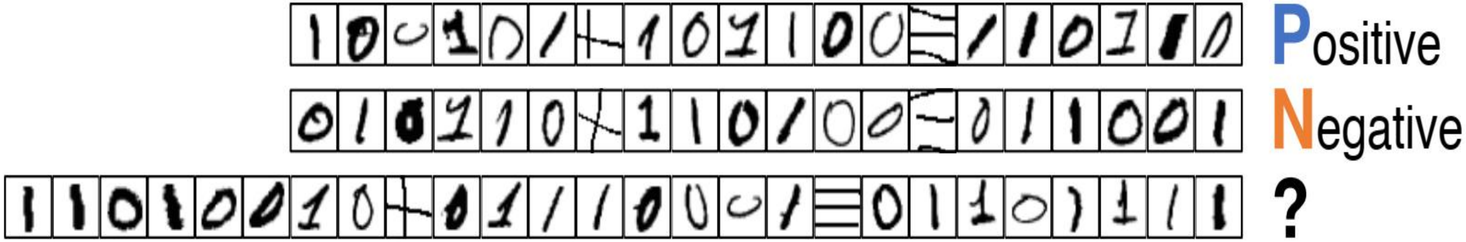


Empirical evaluation

- Benchmarks from [6], [2] and chess scenario.
- Competitors: DeepProbLog [2], ABL [12] and NeurASP [13].







State of the art

- **DeepProbLog.**
  - Reduces the problem to learning the parameters of probabilistic logic programs.
- **NeurASP**
  - Reduces the problem to learning the parameters of probabilistic answer set pro grams.
- **ABL**
  - Computes the neural predictions for each element.
  - Obscures subsets of the neural predictions.
  - Abduces the obscured predictions so that the resulting predictions are consiste nt with the background knowledge.
  - Trains the neural component using obscured and abduced neural predictions.

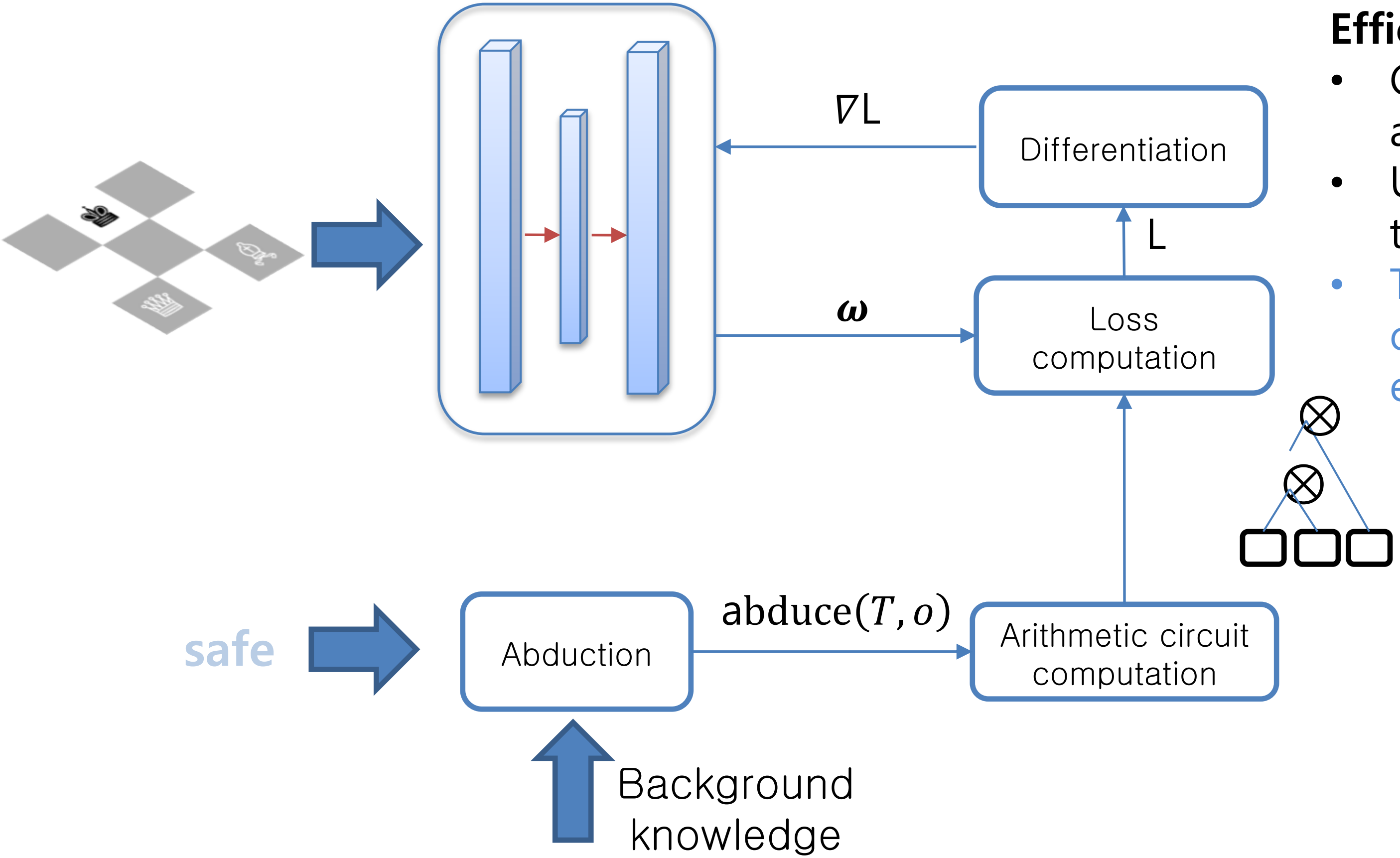
Empirical evaluation

	ADD2x2	OPERATOR2x2	APPLY2x2	DBA(5)	MATH(3)	MATH(5)
accur % NLOG	91.7 ± 0.7	90.8 ± 0.8	100 ± 0	95.0 ± 0.2	95.0 ± 1.2	92.2 ± 0.9
accur % DLOG	88.4 ± 2.5	86.9 ± 1.0	100 ± 0	95.6 ± 1.8	93.4 ± 1.4	timeout
accur % ABL	75.5 ± 34	timeout	88.9 ± 13.1	79 ± 12.8	69.7 ± 6.2	6.1 ± 2.8
accur % NASP	89.5 ± 1.8	timeout	76.5 ± 0.1	94.8 ± 1.8	27.5 ± 34	18.2 ± 33.5
time (s) NLOG	531 ± 12	565 ± 36	228 ± 11	307 ± 51	472 ± 15	900 ± 71
time (s) DLOG	1035 ± 71	8982 ± 69	586 ± 9	4203 ± 8	1649 ± 301	timeout
time (s) ABL	1524 ± 100	timeout	1668 ± 30	1904 ± 92	1903 ± 17	2440 ± 13
time (s) NASP	356 ± 4	timeout	454 ± 652	193 ± 2	125 ± 6	217 ± 3

	PATH(4)	PATH(6)	MEMBER(3)	MEMBER(5)	CHESS-BSV(3)	CHESS-ISK(3)	CHESS-NGA(3)
accur % NLOG	97.4 ± 1.4	97.2 ± 1.1	96.9 ± 0.4	95.4 ± 1.2	94.1 ± 0.8	93.9 ± 1.0	92.7 ± 1.6
accur % DLOG	timeout	timeout	96.3 ± 0.3	timeout	n/a	n/a	n/a
accur % ABL	timeout	timeout	55.3 ± 3.9	49.0 ± 0.1	0.3 ± 0.2	44.3 ± 7.1	n/a
accur % NASP	timeout	timeout	94.8 ± 1.3	timeout	timeout	19.7 ± 6.3	n/a
time (s) NLOG	958 ± 89	2576 ± 14	333 ± 23	408 ± 18	3576 ± 28	964 ± 15	2189 ± 86
time (s) DLOG	timeout	timeout	2218 ± 211	timeout	n/a	n/a	n/a
time (s) ABL	timeout	timeout	1392 ± 8	1862 ± 28	9436 ± 169	7527 ± 322	n/a
time (s) NASP	timeout	timeout	325 ± 3	timeout	timeout	787 ± 307	n/a

Results using 3000 training samples and 3 epochs.

NeuroLog: efficient caching mechanism

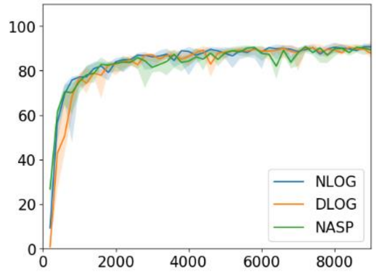


Efficient caching:

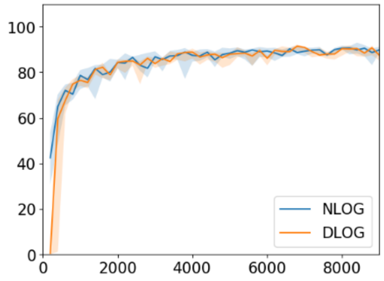
- Compute an circuit for e ach abductive formula.
- Use the compute circuit to compute the loss.
- The number of different circuits equals the numb er of different labels.

NeuroLog vs DeepProbLog and NeurASP

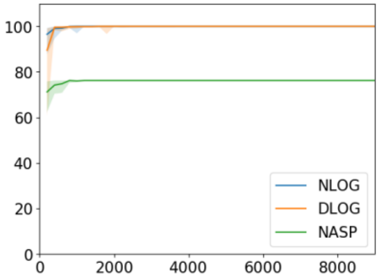
ADD2x2



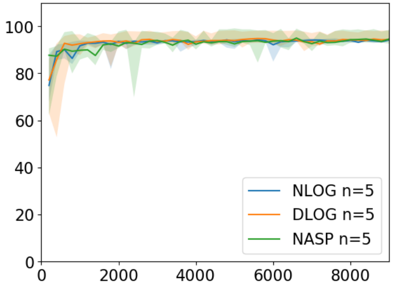
OPERATOR2x2



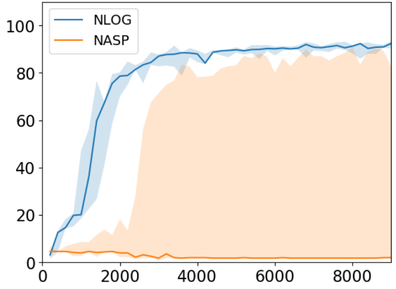
APPLY2x2



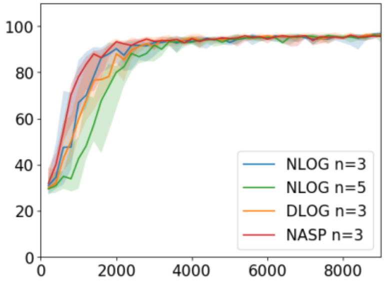
DBA(n)



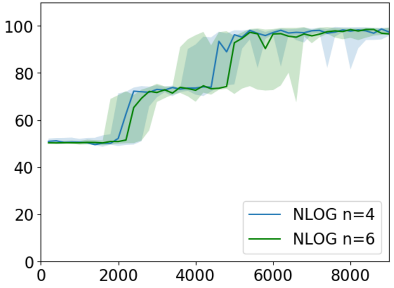
MATH(5)



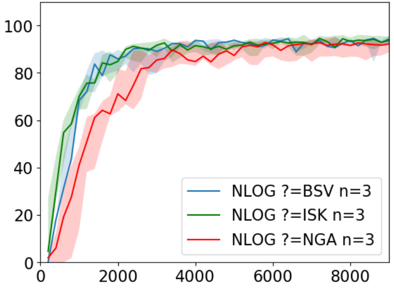
MEMBER(n)



PATH(n)



CHESS-?(n)



Results using 3000 training samples and 3 epochs.

NeuroLog vs ABL

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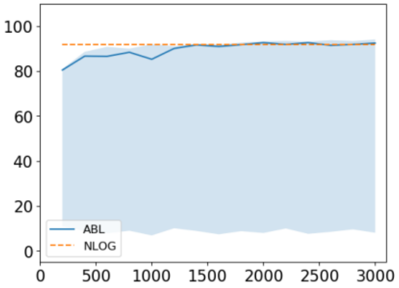
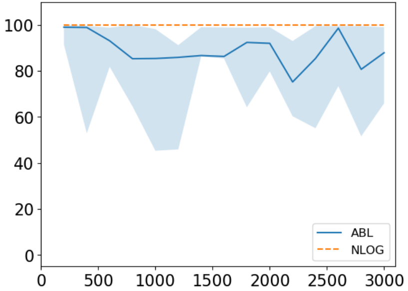
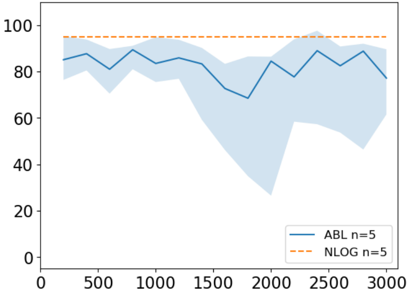
ADD2x2



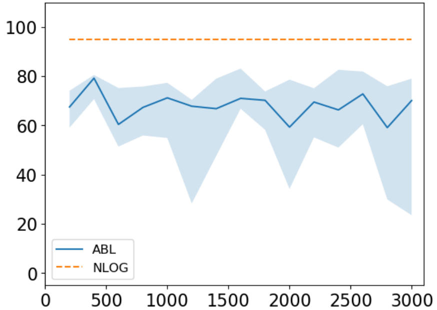
APPLY2x2



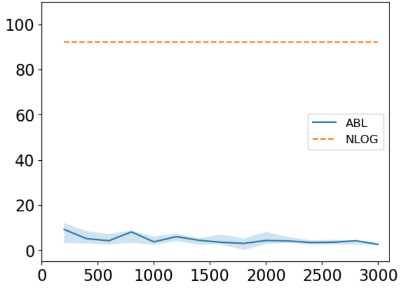
DBA(n)



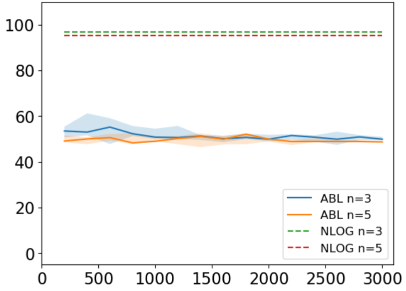
MATH(3)



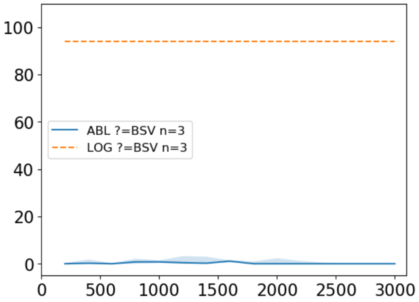
MATH(5)



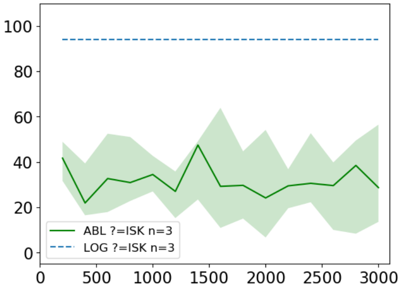
MEMBER(n)



CHESS-BSV(n)



CHESS-ISK(n)



Results using 3000 training samples and 3 epochs.

Summary

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- Compositional: users can plug in nets and logic theories of interest, e.g., non-monotonic, probabilistic, action.
- Natural integration of neural predictions during the training process.
- Outperforms state of the art in terms of training time and efficiency.

References

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